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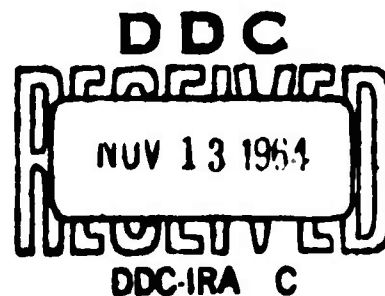
MEMORANDUM
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ADAPTIVE SEGMENTAL DIFFERENTIAL APPROXIMATION

Richard Bellman and Brian Gluss

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Richard Bellman and Brian Gluss

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PREFACE

In previous Memoranda, the problem was considered of observing the output of a black box comprising a complex differential system, determining the nature of the system and its subcircuits, and estimating when the box switches from one subcircuit to another, and which one is operating at any time. In all this work, no account was taken of previous information; we examine here the computational reduction that may be achieved by using such information. The method used has a built-in quality-control check on its efficiency.

Applicability of the mathematical concepts discussed in this Memorandum would be of particular interest to control engineers.

SUMMARY

In two recent papers, we have used a blend of dynamic programming, quasilinearization, and differential approximation methods, which we call segmental differential approximation, in order to estimate the parameters of the differential subcircuits of a black box and the times of switching from one to another, using only the information provided by the output function, either at sample points or continuously.

We discuss here the extension of these results to an adaptive procedure, in which the computations are reduced as the number of switchings that are occurring becomes large, relative to the number of subcircuits present in the system. In the method used, there is an innate quality control on the approximation.

The method may be further refined for electrical systems that are known to be linear.

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ADAPTIVE SEGMENTAL DIFFERENTIAL APPROXIMATION

1. INTRODUCTION

Suppose that it is desired to observe the output $f(t)$ of a black box comprising a differential system with unknown parameters. When no switching occurs, a linear approximation describing the nature of the system may be obtained, using the techniques of differential approximation [1] and quasilinearization [2]. When switching is known to occur periodically from one subcircuit to another, estimates of the times and nature of the switchings, and of the component subcircuits of the black box, may be obtained using segmental differential approximation [3], [4], a technique that comprises a blend of differential approximation, quasilinearization, and dynamic programming.

What the method gives, for $f(t)$ observed in $[0, T]$, is estimates $\tau_1, \tau_2, \dots, \tau_{N-1}$ ($0 \leq \tau_1 \leq \tau_2 \leq \dots \leq \tau_{N-1} \leq T$) of switching times, and differential approximations u^k to $f(t)$ in $[\tau_k, \tau_{k+1})$, where u^k satisfies the linear differential equation

$$(1.1) \quad u_M^k + b_{1k} u_{M-1}^k + b_{2k} u_{M-2}^k + \dots + b_{Mk} u^k = 0,$$

with initial conditions

$$(1.2) \quad u_j^k(\tau_k) = c_{jk}, \quad j = 0, \dots, M-1, \quad k = 0, \dots, N-1,$$

and u_m^k is the m -th derivative of u^k .

These estimates are such that they minimize either S_A or S_B , where

$$(1.3) \quad S_A = \sum_{k=0}^{N-1} \sum_{t_i \in [\tau_k, \tau_{k+1})} [u_0^k(t_i) - f(t_i)]^2,$$

and

$$(1.4) \quad S_B = \sum_{k=0}^{N-1} \left\{ \int_{\tau_k}^{\tau_{k+1}} [u_0^k(t) - f(t)]^2 dt \right\}.$$

Equation (1.3) corresponds to the case in which we are given sample data $f(t_i)$, $0 \leq t_i \leq T$, and equation (1.4) to that in which we observe $f(t)$ continuously in $[0, T]$.

2. ADAPTIVE APPROXIMATION

Suppose now that the black box is observed over a period of time $[0, RT]$ sufficiently long that each subcircuit is used in a large number of subintervals. If we use our previous method, we will obtain RN sets of values of the parameter M -vector $(b_{1k}, b_{2k}, \dots, b_{Mk}) = \underline{b}_k$, $k = 1, \dots, RN$. If there are actually P subcircuits e_1, \dots, e_P , then there will be $n(1), n(2), \dots, n(P)$ values of \underline{b} associated with them respectively, where

$$(2.1) \quad \sum_{w=1}^P n(w) = RN,$$

and if the approximations are good, the RN points in M -space corresponding to the RN \underline{b} 's will form P clusters, with $n(w)$ points in the w -th cluster corresponding to the subcircuit e_w .

If we then estimate the "centers" $\underline{B}_1, \dots, \underline{B}_P$ of these clusters, and a statistic of spread within each cluster e_w —such as average distance from the center \underline{B}_w —then future decisions as to which subcircuit is in operation may be made by minimizing a criterion function over a finite set of P values corresponding to the P points \underline{B}_w , rather than over the infinity of points in \underline{b} -space in the original method. In other words, we may use the information about the nature of the black box obtained earlier to reduce the problem to a discrimination problem.

The idea will become clearer if we first consider linear differential systems.

2.1. Linear Systems

Let us now assume that the black box is known to be a linear differential system, consisting of P (P unknown) linear differential subcircuits of orders less than or equal to M . Provided that N is larger than the maximum number of switchings per time-interval T , the segmental differential approximation method described in

[3] and [4] will in fact give exact fits; i.e., the minimum values of S_A or S_B obtained will be zero.*

In this case, then, the P clusters in \underline{b} -space will have zero radius, $n(w)$ values \underline{B}_w being obtained, $w = 1, \dots, P$. After this large number RN of \underline{b} 's has produced the P points \underline{B}_w , from then on we compare the present output with them to determine which subcircuit w is operating. For example, in the interval of time $[RT, (R + 1)T)$, confining ourselves to the continuous data case, we proceed as follows.

First, we perform some computations associated with the \underline{B}_w that we shall require in all future time-interval analysis: for each \underline{B}_w we obtain M independent solutions $u^w(t)$ of the differential equation associated with \underline{B}_w . Then, in equation (1.4), $u_0^k(t)$ must be one of the $u^w(t)$. Referring back to [4], if $F_N[(R + 1)T]$ is the minimum value of S_B (i.e., zero), then we have the functional equation

$$(2.2) \quad F_N[(R + 1)T] = \min_{\tau} [F_{N-1}(\tau) + S_B(\tau, (R + 1)T)],$$

$$\text{where } RT \leq \tau \leq (R + 1)T,$$

and

* If $S \neq 0$, this implies either that M is smaller than the maximum subcircuit order, or that N is smaller than the number of switchings, or both.

$$(2.3) \quad S_B(\tau, (R+1)T) = \min_w \int_{\tau}^{(R+1)T} [u^w(t) - f(t)]^2 dt,$$

entailing a discrete finite search. Note that if there are N switchings in $[RT, (R+1)T)$, then

$F_{N-1}[(R+1)T] \neq 0$, so that equation (2.2) is always meaningful. Since we compute the F_r iteratively, if r_1 is the first—and lowest— r for which $F_r[(R+1)T] = 0$, we then know that there have been r_1 switchings in the time-interval. If $r_1 > N$, either our choice of N was not large enough—which we have hypothesized not to be the case—or what is discussed below occurs.

2.1.1. Quality Control. If $F_N[(R+1)T]$ turns out to be nonzero, this implies that a subcircuit has been in use during the time-interval $[RT, (R+1)T)$ that has not been used before. In other words, our estimate P is in error, and there is in fact a further \underline{B}_w to be computed. For this, we have to return to the original method.

What in effect is happening is that we have a nice built-in indicator F_N in the method that shows when a subcircuit is used for the first time: in this case we simply go back to the method of [3] and [4], so that at the very worst we make the same computations as in the original method, and usually are much better off.

2.2. Nonlinear Systems

If we apply the same procedure to approximate identification of nonlinear systems with unknown parameters but known parametric type, then the problem is conceptually much as before.

If the type of system is unknown, and we use linear differential approximation, then the \underline{B}_w cluster will not have zero radius, and statistical methods of estimation must be introduced.

Such a method, for example, would, roughly speaking, be as follows:

(a) Identify and separate the clusters using some discriminating procedure.

(b) Determine for each cluster a point \underline{B}_w such that the maximum distance r_w of any point in the cluster from \underline{B}_w is minimized.

(c) Proceed as in the linear case, where F_N will now not be zero.

(d) If $F_N[(R + 1)T]$ is larger than a prescribed error criterion ϵ , re-estimate the \underline{B}_w and/or P .

3. CONCLUSIONS

We have considered above an important new method for solving the "inverse problem" of identifying the nature of black boxes from their outputs. When the box is known

to have differential characterizations of known parametric types, where the values of the parameters are unknown, the method is exact and computationally economical.

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